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ABSTRACT

This teacher's guide for a semester course in analytic geometry is based on the text "Analytic Geometry" by W. K. Morrill. Included is a daily schedule of suggested topics and homework assignments. Specific teaching hints are also given. The content of the course includes point and plane vectors, straight lines, point and space vectors, planes, straight lines in space, circles, conics, transformation of axes, and polar coordinates. (Author/CT)

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BALTIMORE COUNTY PUBLIC SCHOOLS

A Tentative Guide

# ANALYTIC GEOMETRY

TOWSON, MARYLAND

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A Tentative Guide

# ANALYTIC GEOMETRY


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Towson, Maryland
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# ANALYTIC GEOMETRY

Analytic Geometry is a second semester course in the Trig-Analytics sequence. It is designed for those students who have successfully completed Algebra II, Geometry and Trigonometry. This course is also prerequisite for those students desiring the Calculus in the 12th year.

The text for the course is Analytic Geometry by W. K. Morrill which utilizes a vector approach to the subject. However, the use of vector techniques in no way implies that this should become a course in vector algebra. The page references are keyed to both the old and revised editions.

The guide is tentative and subject to continuous teacher evaluation. The time allotment, sequence of topics and homework assignments are suggestive. The teacher may adjust these according to the needs of the students. For instance, some teachers may wish to interchange the units on Transformation of Axes and Polar Coordinates. Teachers will want to avail themselves of appropriate overhead transparencies in the Geometry and Calculus series, which may be found in the department files. Standardized tests are also available at the Central Testing Office. These may be obtained through your department chairman.

The Board of Education and the Superintendent of Schools wish to express their appreciation to the Curriculum Committee and to all mathematics teachers of Baltimore County whose efforts made possible the development of this curriculum guide.

Special commendation is due Mrs. Betty Hagedorn, secretary to the committee, for her careful and painstaking efforts in the production of this guide.

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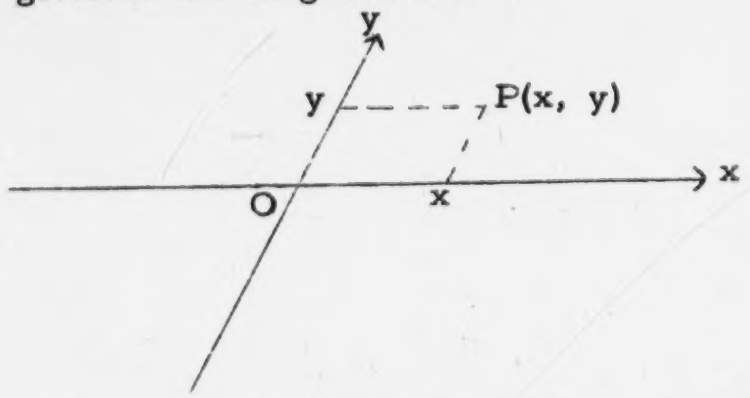
| | **UNIT ONE** | | |
| | **The Point and Plane Vectors** | | |
| 1 | Introduction (2-1) | MRev... | 19 |
| | | M..... | 18 |
| | Emphasize the fundamental concept of analytic geometry: a point is a member of the graph of a relation if and only if its coordinates satisfy the equation of the relation. | | |
| | The Rectangular Coordinate System | MRev... | 19-21 |
| | | M..... | 18-20 |
| | Note that the choice of a <u>rectangular</u> system of coordinates is arbitrary. A different system is often more useful for the solution of specific problems. An example of an alternate system of axes is given in the diagram below. | | |




THE OBLIQUE SYSTEM OF COORDINATES

Emphasize the differences between a right-hand coordinate system and a left-hand coordinate system. Note that P(x, y) represents <u>any</u> point in the plane and not only those in quadrant I.

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| 1 (cont'd) | Symmetry (2-3)<br>Symmetry with respect to a point<br>Symmetry with respect to a line<br>SUGGESTED BASIC ASSIGNMENT:<br>The problems in the Suggested Basic Assignment are not meant to constitute the entire assignment. They are problems which should be included in assignment.<br>MRev... Pp. 21, 22: Ex. 3, 6, 8, 10, 13, 18, 20, 23, 25, 26<br>M..... Pp. 20, 21: Ex. 3, 6, 8, 10, 13<br>MRev... P. 23: Ex. 2<br>M..... Pp. 21, 22: Ex. 2 | MRev... 22-23<br>M..... 21 | |
| 2 | Projections<br>Students who have studied the SMSG Geometry consider the projection of a point or segment in a line to be a set of points. The text defines the projection of a directed segment $P_1P_2$ in a line L to be the directed distance $\overline{AB}$ where A is the projection of $P_1$ in L and B is the projection of $P_2$ in L. Observe that in SMSG Geometry $\overline{AB}$ represents a segment with A and B as end points and AB represents the undirected length of $\overline{AB}$. However, note that in the text all segments are directed segments. $P_1P_2$ represents the directed segment from $P_1$ to $P_2$; $\overline{AB}$ represents the directed distance from A to B where A and B are points in the x or y axes, and $\lvert P_1P_2 \rvert$ represents the length of $P_1P_2$.<br>Indicate the differences between the notation used in SMSG and the text to avoid confusion. | MRev... 23-25<br>M..... 22-24 | |

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| 2 (cont'd) | Scalar Components of a Segment (2-5)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 25: Ex. 1, 3<br>MRev... P. 27: Ex. 1, 3, 4<br>M..... Pp. 23, 24: Ex. 1, 3<br>M..... Pp.25, 26: Ex. 1, 3, 4 | MRev... 26-27<br>M..... 24-25 | |
| 3 | Distance Between Two Points (2-6)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 29, 30: Ex. 2a, 3, 6, 7, 8, 9, 10, 12, 13<br>M..... P. 28: Ex. 2a, 3, 6, 7, 8 | MRev... 27-30<br>M..... 26-28 | |
| 4 | Direction Cosines of a Segment (2-7).<br>Note that the text does not define direction angles of a segment. This should be done as follows: The direction angles of $\overline{P_1P_2}$ where $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are the angles measured in a counterclockwise direction from two rays $\overrightarrow{P_1A}$ and $\overrightarrow{P_1B}$ parallel to the positive x and y axes respectively to the directed segment $P_1P_2$. These angles are denoted by $\alpha$ and $\beta$ respectively. Illustrate these definitions with diagrams similar to the ones below. | MRev... 31-33<br>M..... 29-31 | |

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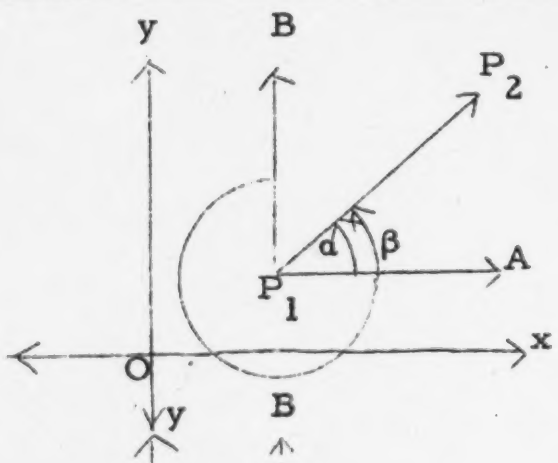
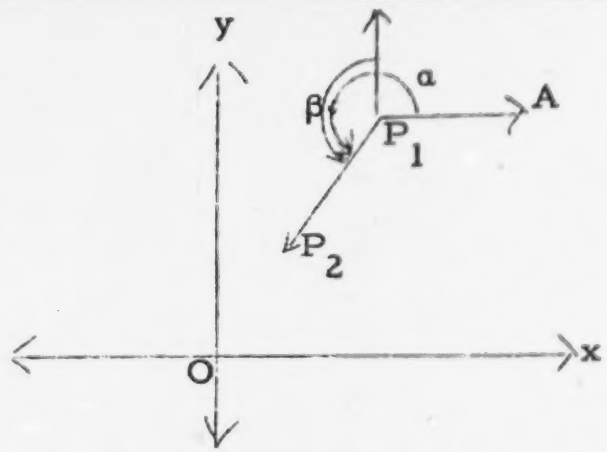
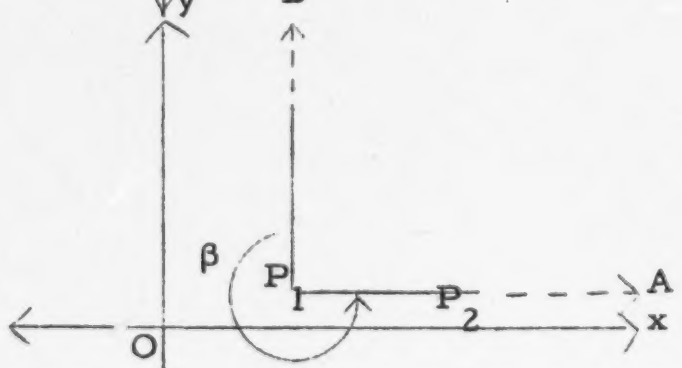
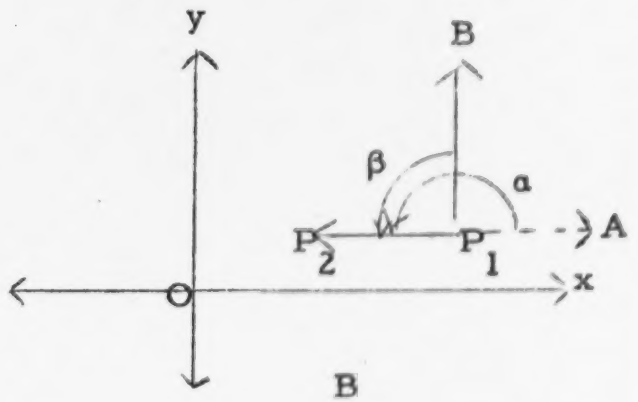
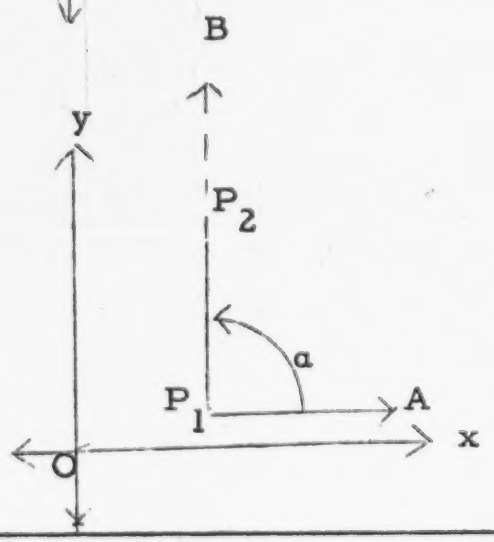
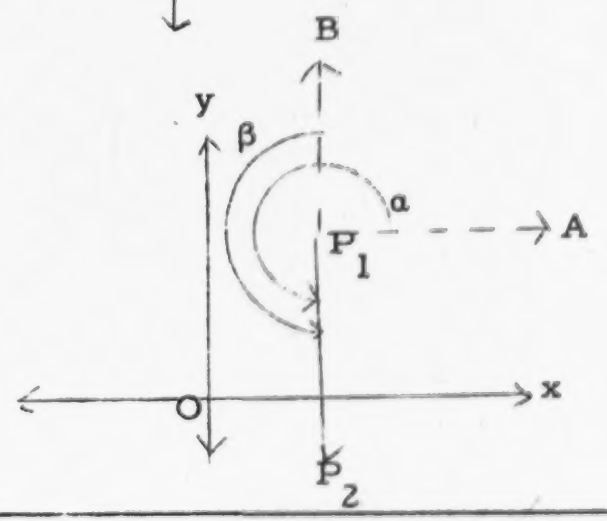
| 4 (cont'd) |             | | |

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| 4 (cont'd) | The direction cosines, l and m, of $P_1P_2$ are defined to be the cosines of $\alpha$ and $\beta$ respectively. Emphasize the advantages of the cosine function over the other trigonometric functions.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 32, 33: Ex. 1, 2, 3, 4, 5, 8<br>M..... Pp. 30, 31: Ex. 1, 2, 3, 4, 5, 8 | | |
| 5 | Plane Vectors (2-8)<br><br>Vectors are central in the development of the text. They are abstract concepts, being sets of equivalent directed segments. This is similar to the idea of equivalence classes. The analogy should be pursued with students familiar with the concept. Students of physics are familiar with the vector concept. However, vectors have been defined for them as "quantities possessing magnitude and direction". This is a geometrical interpretation and is what our text means by a representative of the vector. A <u>scalar</u> is a real number. Examples of scalars are length, area, volume, time, mass, and temperature. Examples of physical quantities that are represented by vectors are force, velocity, and acceleration. Indicate the similarity between the algebra of real numbers and the algebra of vectors. <u>However, do not prove all the algebraic properties of vectors.</u> The purpose of this course is to develop analytic geometry, not vector algebra.<br><br>EXAMPLE: Express [ 3, 4 ] in terms of [ 1, 0 ] and [ 0, 1 ].<br><br>SOLUTION: [ 3, 4 ] = [ 3, 0 ] + [ 0, 4 ] = 3[ 1, 0 ] + 4[ 0, 1 ]. | MRev... 33-36<br>M..... 31-34 | |

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| 5 (cont'd) | Several examples like this will lead to the generalization that the vectors [ 1, 0 ] and [ 0, 1 ] are the "building blocks" of the system of vectors.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 36, 37: Ex. 1, 2a, 3, 4, 5, 6, 10, 11, 12<br>Omit number 8.<br>M..... P. 34: Ex. 1, 2a, 3, 4, 5, 6, 10(and 11, 12 of MRev.) | | |
| 6 | Angle Between Two Vectors (2-9)<br><br>An alternate proof of the formula for determining the angle between two vectors is given below and may be used if the teacher prefers it to the one in the text.<br><br>THEOREM: If $\vec{u} = [ u_1, u_2 ]$ and $\vec{v} = [ v_1, v_2 ]$ are two non-zero vectors, the angle $\theta$ between them is given by<br><br>$$\cos \theta = \frac{u_1 v_1 + u_2 v_2}{|\vec{u}|\ |\vec{v}|}$$<br><br>PROOF: Let $\vec{w} = \vec{v} - \vec{u}$. Thus, $\vec{w} = [ v_1 - u_1, v_2 - u_2 ]$<br>By the law of cosines (see diagram),<br>$|\vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}|\ |\vec{v}| \cos \theta$ | MRev... 37-39<br>M..... 34-36 | |

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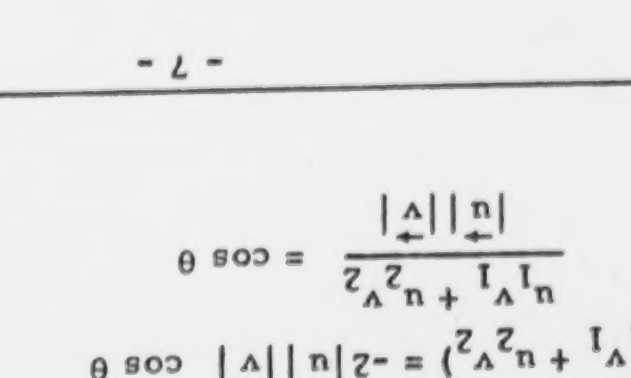
| 6 (cont'd) | $\vec{v} = [\, v_1, v_2 \,]$   But, since $\vec{w} = [\, v_1 - u_1, v_2 - u_2 \,]$, $\|[\, v_1 - u_1, v_2 - u_2 \,]\|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$ $(v_1 - u_1)^2 + (v_2 - u_2)^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$ $(v_1^2 - 2v_1u_1 + u_1^2) + (v_2^2 - 2v_2u_2 + u_2^2) = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$ $(v_1^2 + v_2^2)+(u_1^2 + u_2^2) - 2(u_1v_1 + u_2v_2) = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$ $|\vec{v}|^2 + |\vec{u}|^2 - 2(u_1v_1 + u_2v_2) = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\theta$ $-2(u_1v_1 + u_2v_2) = -2|\vec{u}||\vec{v}| \cos\theta$ $\dfrac{u_1v_1 + u_2v_2}{|\vec{u}||\vec{v}|} = \cos\theta$ Q. E. D. | | |

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| 6 (cont'd) | An elegant feature of the above proof is that there is no reference to any coordinate system and thus the proof generalizes to 3-space quite easily.<br><br>The teacher should note that the text's definition of the dot product is different from the one in PSSC Physics. It is suggested that the teacher consult the teacher of physics to avoid confusion in the term dot product.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 41, 42: Ex. 8, 16, 17, 18<br>M..... Pp. 38, 39: Ex. 8 (and 16, 17, 18 of MRev.) | | |
| 7 | Parallel and Perpendicular Vectors (2-10)<br><br>The teacher should study the concept of proportionality in Chapter 7, Similarity, of SMSG Geometry with Coordinates. This will clear up the apparent possibility of division by zero in the discussion on parallelism of vectors on page 40 of the text. To avoid this difficulty, the teacher may want to adopt the results of problem 9 on page 41 of MRev as the definition of parallel vectors.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 41, 42: Ex. 1a, 2, 5, 6, 9, 10, 11, 12, 14a, 19<br>M..... Pp. 38, 39: Ex. 1a, 2, 5, 6, 7 (and 9, 10, 11, 12, 14a, 19 of MRev.) | | MRev... 39-42<br>M..... 36-38 |

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| 8 | The Coordinates of a Point That Divides a Segment in a Given Ratio (2-11)<br>The teacher should emphasize the solution of each problem by using an appropriate vector equation rather than by substitution in the formulas on page 44 of the text. In effect, each problem should be a redevelopment of the formulas on page 44.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 46, 47: Ex. 1, 3, 15, 16, 17, 18, 19<br>M..... Pp. 43, 44: Ex. 1, 3, 15, 16, 17, 18, 19 | MRev...<br>M..... | 42-45<br>39-42 |
| 9 | The Midpoint of a Segment and Analytic Proofs of Geometric Theorems (2-12)<br>The teacher should spend most of the lesson solving problems such as numbers 9, 11, 13, and 14 on pages 46 and 47 of the text.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 46, 47: Ex. 5, 7, 9, 10<br>M..... Pp. 43, 44: Ex. 5, 7, 9, 10 | MRev...<br>M..... | 45-47<br>42-44 |
| 10 | Complementary Vectors (2-13)<br>Area of a Triangle and the Bar Product of Two Vectors (2-14)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 50, 51: Ex. 1, 2, 3, 4b, 5a, 6, 7<br>Omit number 9 for students not familiar with determinants.<br>M..... Pp. 47, 48: Ex. 1, 2, 3, 4b, 5a, 6, 7 | MRev...<br>M.....<br>MRev...<br>M..... | 48<br>44-45<br>49-51<br>45-47 |

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| 11 | Review for Test and Review Exercises<br><br>Be sure to include problems from the Review Exercises at the end of each chapter of the text. Besides providing good problems for review, material is often introduced or extended beyond the point covered earlier in the chapter. | MRev... | 52-54<br>48-49 |
| 12 | Test on Chapter 2 | M..... | |

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| 13 | **UNIT TWO**<br>**The Straight Line**<br>**Direction Numbers and Direction Cosines of a Line (3-1)**<br>Emphasize that a line has an infinite number of pairs of direction numbers associated with it, while it has only two pairs of associated direction cosines.<br>EXAMPLE: The line $\overleftrightarrow{AB}$, with A(1, 2) B(-5, 6) has direction numbers: [ -6, 4 ], [ -3, 2 ], [ 3, -2 ], [ 9, -6 ] etc.<br>The direction cosines of $\overleftrightarrow{AB}$ are:<br>$\left[\frac{-6}{2\sqrt{13}}, \frac{4}{2\sqrt{13}}\right] = \left[\frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right]$<br>$\left[\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right] = \left[\frac{9}{3\sqrt{13}}, \frac{-6}{3\sqrt{13}}\right]$ | MRev... 55-56<br>M..... 50-51 | |
| | **The Slope of a Line (3-2)**<br>Although slope is not a new concept to the student, inclination is. Inclination in the text is used in two contexts, the angle between the positively directed x axis and the line, measured in a counter-clockwise direction and as the measure of that angle.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 58: Ex. 1ab, 2, 3, 4, 5, 6ab<br>M..... P. 54: Ex. 1ab, 2, 3, 4, 5, 6ab | MRev... 57-58<br>M..... 51-54 | |

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| 14 | **Parametric Equations of Lines (3-3)**<br>This concept is not new to those students who have studied SMSG Geometry. The vector approach of this text, however, should make for a better understanding. For students who did not have SMSG Geometry parametric equations will be a new concept. Students will need this method of writing linear equations in three dimensions so they should become adept in using this method now. Have students graph some lines whose equations are given in parametric form. (Exercise 2 at end of section.) | MRev... | 59-61<br>M..... 54-56 |
| 15 | **Equations of Lines in Direction Number Form (Symmetric Form) (3-4)**<br>$x = x_1 + t\Delta x$ Parametric<br>$y = y_1 + t\Delta y$ Form<br>$\frac{x - x_1}{\Delta x} = \frac{y - y_1}{\Delta y}$ Symmetric Form<br>As can be seen above the symmetric form of an equation of a line is easily derived from the parametric form by solving for the parameter. Symmetric form is not usable if either $\Delta x$ or $\Delta y$ is 0. Give students practice in converting from parametric form to symmetric form. Have the students do some of the exercises at the end of this section in symmetric form and leave their answers in that form. Rule 3-13 should be omitted.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 61, 62: Ex. 1 (in parametric and symmetric form), 2, 6ab, 9ab<br>M..... P. 56: Ex. 1 (in parametric and symmetric form), 2, and include 6ab, 9ab, of MRev. | MRev... | 59-64<br>M..... 54-56 |

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| 16 | **The General Linear Equation $ax + by + c = 0$ (3-5)**<br>Have the students thoroughly understand the two part proof of:<br>a) The set of points P(x, y) having the property that $P_1P$ $[x - x_1, y - y_1]$ is perpendicular to $\vec{u}$ $[a, b]$ containing $P_1$ has an equation $ax + by = c$.<br>b) $ax + by = c$ is an equation of a line which has a vector $[a, b]$ perpendicular to it.<br>This section is an exciting and powerful result of our previous vector treatment.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 64, 65: Ex. 2ab, 3ab, 6ab<br>M..... P. 59: Ex. 2ab, 3ab, (and 6ab of MRev.) | MRev... 65-68<br>M..... 59-63 | |
| 17 | **Lines Perpendicular to and Parallel to a Given Line (3-11)**<br>This section and the exercises which follow give further emphasis to Section (3-5).<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 81, 82: Ex. 1abcd, 2ab, 3ab, 4ab<br>M..... Pp. 73, 74: Ex. 1abcd, 2ab, 3ab, 4ab | MRev... 80-82<br>M..... 72-73 | |

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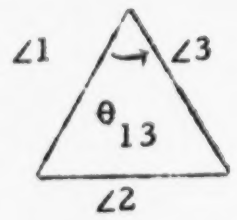
| 18 | **Angle Between Two Lines (3-6)**<br><br>The teacher should give equal emphasis to finding the angle between two lines through the cosine formula and through the tangent formula. The cosine method is already familiar to the students from Chapter 2 and may be preferred by them. However, the tangent method should be encouraged as it reinforces the inclination concept and is useful in the calculus. The teacher should encourage drawings from students and give them exercises where a specified angle's cosine or tangent is required. (Problems 8, 10, 11, 12 at end of this section.) A good memory technique for finding the tangent of an angle is:<br><br>  <br><br>$\text{Tan } \theta_{13} = \frac{s_3 - s_1}{1 + s_1 s_3}$<br><br>Note that the symbol "$\theta_{13}$" means the directed angle between $\angle_1$ and $\angle_3$.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 72, 73: Ex. 1a, 3a, 5, 6a, 8, 10, 11, 12<br>M..... P. 65: Ex. 1a, 3a, 5, 6a, 8 (and 10, 11, 12 of MRev.) | MRev... 70-71<br>M..... 63-65 | |
| 19 | **Point Slope Form of Equation of Line (3-7)** | MRev... 73-75<br>M..... 65-68 | |

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| 19 (cont'd) | **Intercept Form of Equation of Line (3-8)**<br><br>**Slope, Intercept Form of Equation of a Line (3-9)**<br><br>**The three forms above should be a review for the students.**<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 78, 79: Ex. 1abc, 2, 4, 13, 18, 19<br>M..... Pp. 71, 72: Ex. 1abc, 2, 4 (and 13, 18, 19 of MRev.) | | |
| 20 | **Distance from Line to Point (3-12)**<br><br>**The teacher should emphasize:**<br><br>**a. If the number** $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} > 0$, **then the point** $P(x_1, y_1)$ **lies on the side of the line** $ax + by + c = 0$ **toward which the coefficient vector** $[a, b]$ **points, provided** $a > 0$.<br><br>**b. If the number** $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} < 0$, **then the point** $P(x_1, y_1)$ **lies on the side of the line opposite to that which the coefficient vector points.**<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 85: Ex. 3a, 4a, 5a, 8, 9<br>M..... Pp. 76, 77: Ex. 3a, 4a, 5a (and 8, 9 of MRev.) | MRev... 83-85<br>M..... 74-76 | |

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| 21 | **Intersecting Lines (3-13)**<br><br>**This section gives an opportunity to introduce the class to determinants; evaluation of a second order determinant and its use for finding the point of intersection of 2 lines. If the class finds this easy or a review, it is advisable to evaluate third order determinants as they will be useful in the chapters which follow. The diagonal method of expansion may be used in 2nd order and 3rd order determinants. Expansion by cofactors is necessary for the expansion of all determinants beyond 3rd order. Encourage students to use determinants in the solution of problems 3, 4, 5 at the end of this section.**<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 90: Ex. 3, 4, 5<br>M..... P. 80: Ex. 3, 4, 5 | MRev... 86-88<br>M..... 78-80 | |
| 22 | **Bisectors of Angles (3-15)**<br><br>**The set of points equidistant from the sides of an angle is a ray which bisects the angle. The teacher should have the students adopt this "locus" theorem to their work in Section 3-15. Special care should be given to making a drawing and understanding which sign should be used in the problem at hand from the general equation for the angle bisector**<br><br>$$\frac{a_1x + b_1y + c}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$ | MRev... 91-93<br>M..... 81-83 | |

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| 22 (cont'd) | Note: Illustrative example 3-26 should read: Find an equation of the bisector of the angle such that:<br>1) the angle is formed by the lines $l_1 = 3x - 4y + 10 = 0$ and $l_2 = 4x - 3y + 12 = 0$<br>2) the origin is an interior point of the angle.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 93: Ex. 1, 3, 6, 8, 10<br>M..... P. 83: Ex. 1, 3 (and 6, 8, 10 of MRev.) | | |
| 23 | Review<br>Since Sections 3-10, 3-14, 3-16, 3-17 have been omitted, review exercises 10, 19, 20, 30, 35, 36 should be omitted. | | |
| 24 | Test | | |

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| | **UNIT THREE** | | |
| | **The Point and Space Vectors** | | |
| 25 | The Point in Space (9-1) | MRev... | 297-298 |
| | | M..... | 277-278 |
| | Projections (9-2) | MRev... | 298-299 |
| | | M..... | 278-279 |
| | Note the dual usage of "projection of a segment" as meaning both a segment and a directed distance. | | |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 300: Ex. 2, 5ae, 7, 9, 10<br>M..... P. 280: Ex. 2, 5ae, 7, 9, 10 | | |
| 26 | Scalar Components of a Segment (9-3) | MRev... | 301 |
| | | M..... | 281 |
| | Length, or Magnitude, of a Segment (9-4) | MRev... | 301-303 |
| | | M..... | 281-283 |
| | Direction Cosines of a Segment (9-5) | MRev... | 303-304 |
| | | M..... | 283-284 |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... Pp. 304,305: Ex. 1a, 5a, 6, 7, 10<br>M..... Pp. 284,285: Ex. 1a, 5a, 6, 7, 10 | | |
| 27 | Space Vectors (9-6) | MRev... | 305-309 |
| | | M..... | 285-288 |

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| 27 (cont'd) | Cosine of the Angle Between Two Vectors (9-7) | MRev... | 309-312 |
| | | M..... | 288-291 |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 308: Ex. 3, 4ac, 6a, 7, 10a<br>M..... P. 288: Ex. 3, 4ac, 6a, 7, 10a<br>MRev... P. 312: Ex. 1, 2a, 3a, 9<br>M..... Pp. 291-292: Ex. 1, 2a, 3a, 9 | | |
| 28 | The Coordinates of a Point that Divides a Segment in a Given Ratio (9-8) | MRev... | 312-314 |
| | | M..... | 292-294 |
| | The Midpoint of a Segment (9-9) | MRev... | 314-315 |
| | | M..... | 294 |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 315: Ex. 1, 2, 5, 7<br>M..... Pp. 294-295: Ex. 1, 2, 5, 7 | | |
| 29 | Review | | |
| 30 | Test | | |

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| | **UNIT FOUR** | | |
| | **The Plane** | | |
| 31 | An Equation of a Plane (10-1)<br><br>It should be stressed that the equation for a plane in space is a linear equation in one or more of the variables x, y, z. The general equation of a plane is $Ax + By + Cx + d = 0$. The teacher should derive carefully the concept that for this equation the normal vector [ A, B, C ] is $\perp$ to the plane.<br><br>The teacher should emphasize that:<br>(1) in a one dimensional system<br>$x = 5$<br>is the equation of a point.<br>(2) in a two dimensional system<br>$x = 6$<br>$y = 3$<br>$x + 2y = 12$<br>are equations of lines.<br>(3) in a three dimensional system<br>$x + y = 6$<br>$x + z = 3$<br>$y + z = 9$<br>$x + 3y + 4z = 6$<br>are equations of planes. | MRev... | 318-319<br>M..... 297-298 |

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| | SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 320: Ex. 2, 4, 6, 9, 10, 11, 12<br>M..... P. 299: Ex. 2, 4, 6, 9, 10, 11, 12 | | |
| 32 | Parallel and Perpendicular Planes (10-2)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 322: Ex. 1, 2, 3<br>M..... P. 301: Ex. 1, 2, 3 | MRev... | 321<br>M..... 300 |
| 33 | Intercept Equation of a Plane (10-3)<br>The teacher should give practice in the graphing of an equation by the intercept method.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 323: Ex. 1, 2<br>M..... P. 302: Ex. 1, 2 | MRev... | 322-323<br>M..... 301-302 |
| 34 - 5 | Vector Product of Two Vectors (10-4)<br>The teacher should stress:<br>(1) the vector product is a vector and not a number<br>(2) $\vec{u} \times \vec{v}$ is perpendicular to both u and v<br>(3) $\vec{u} \times \vec{v}$ is the normal of the plane containing $\vec{u}$ and $\vec{v}$.<br>The teacher should develop carefully the theorem on page 324 (304) and be sure to include the area of parallelograms. | MRev... | 323-326<br>M..... 302 |

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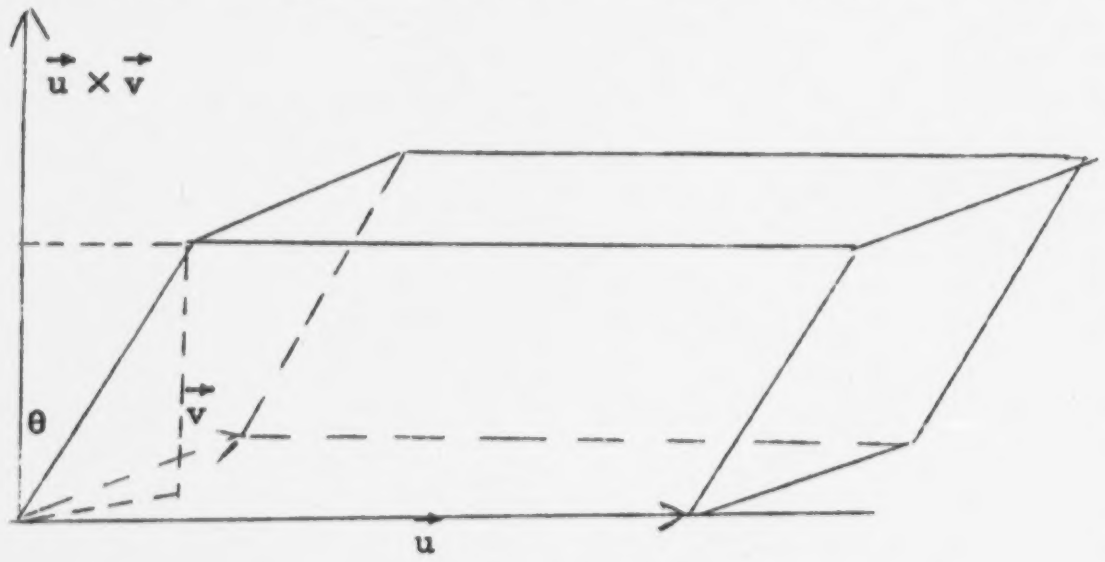
| 36 | **Alternating Product or Scalar Triple Product of Three Vectors (10-5)**<br>**The teacher should prove in class Example 8, page 327. (M. . . p. 305)** | MRev. . . 326<br>M. . . . . 305 | |




Additional properties of the scalar triple product are:

(1) $\vec{u}$, $\vec{v}$, and $\vec{w}$ are coplanar if and only if $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$

(2) the value of the scalar triple product is not changed by a cyclic permutation of the elements which form the product. $\vec{u} \cdot (\vec{v} \times \vec{u}) = \vec{w} \cdot (\vec{u} \times \vec{v}) = \vec{v} \cdot (\vec{w} \times \vec{u})$

(3) the dot and cross in a scalar triple product can be interchanged without affecting the value of the product. $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$

SUGGESTED BASIC ASSIGNMENT:

MRev. . . Pp. 326, 327: Ex. 2, 4, 5, 6, 7
M. . . . . P. 305: Ex. 2, 4, 5, 6, 7

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| 37 | Determining a Plane Satisfying Three Conditions (10-6)<br>The illustrations in the text are based on the theorem at the bottom of the page 328. An alternate method is to find the cross product of two of the vectors which results in the normal of the plane of these two vectors. Follow Example 10-1 on page 319.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 330: Ex. 1, 2, 3, 4<br>M..... Pp. 307,8: Ex. 1, 2, 3, 4 | MRev...<br>M..... | 328<br>305 |
| 38 | Distance From a Plane to a Point (10-7)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 333: Ex. 1, 2, 3<br>M..... P. 310: Ex. 1, 2, 3 | MRev...<br>M..... | 331<br>308 |
| 39 | Angle Between Two Planes (10-8)<br>The magnitude of the four dihedral angles formed by two intersecting planes are numerically equal, respectively, to the four corresponding angles formed by the two lines that can be drawn through any point in space perpendicular to the given planes. | MRev...<br>M..... | 334<br>310 |

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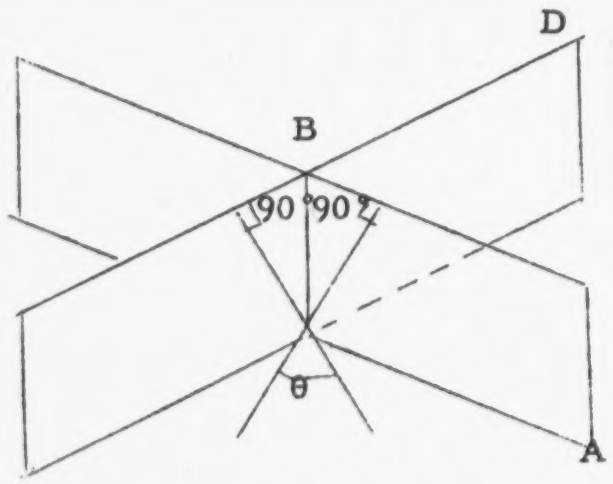
| 39 (cont'd) |  | | |
| | The teacher should stress that the angle formed by two planes is found by computing the angle between the two normals and is similar to the theorem in Geometry which states "two angles whose sides are respectively perpendicular are equal." | | |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 336: Ex. 1<br>M..... P. 312: Ex. 1 | | |
| 40 | Review | | |
| 41 | Test | | |

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| | **UNIT FIVE** | | |
| | **Straight Line in Space** | | |
| 42 | Direction Numbers and Direction Cosines (11-1) | MRev... | 338 |
| | | M..... | 315 |
| | Since the analogy to Sections 9-3 and 9-5 is so very marked, only a brief discussion of this section should be necessary. Nevertheless a discussion should ensue for review and reinforcement purposes. | | |
| | Parametric Equations of a Line (11-2) | MRev... | 340 |
| | | M..... | 317 |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 339: Ex. 1a, 1b, 2a, 3, 4<br>M..... P. 316: Ex. 1a, 1b, 2a, 2b, 3, 4<br>MRev... P. 342: Ex. 1a, 1c, 2a<br>M..... P. 320: Ex. 1a, 1c, 2a | | |
| 43 | Symmetric Equations of a Line (11-3) | MRev... | 341 |
| | | M..... | 318 |
| | The General Equations of a Line (11-4) | MRev... | 341 |
| | | M..... | 318 |
| | The most common error made by students is that of thinking that since a first degree equation in two variables represents a line in two dimensions, then a first degree equation in three variables represents a straight line in space. Great emphasis should be placed on the fact that this is not true. Emphasize that a line in space cannot be described by less than two first degree equations. Indeed, it should be apparent that even when two equations are | | |

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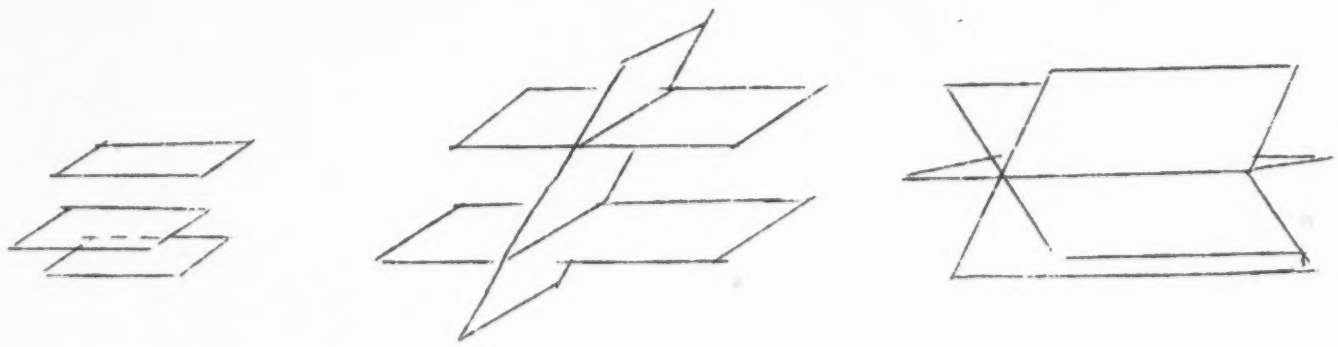
| 43 (cont'd) | used, this does not represent a unique description. Since an infinite number of planes pass through a given line, any two such planes considered together can be used to represent the line.<br><br>Show that the cross product of the coefficient vectors of two planes whose intersection describes the line determines a triple of direction numbers for that line.<br><br>Students should be facile in changing from the general to the symmetric and parametric forms.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 343: Ex. 3ad, 4a, 5, 6, 7, 8, 9, 15b<br>M..... P. 340: Ex. 3ad, 4a, 5, 6, 7, 8, 9, 15b | | |
| 44 | Intersecting Planes (11-5)<br><br>The development and practice in spatial perception is particularly evident in this section. Students should be asked to submit sketches showing the different possibilities in which three planes in space can intersect.<br><br>  | MRev... 345<br>M..... 322 | |

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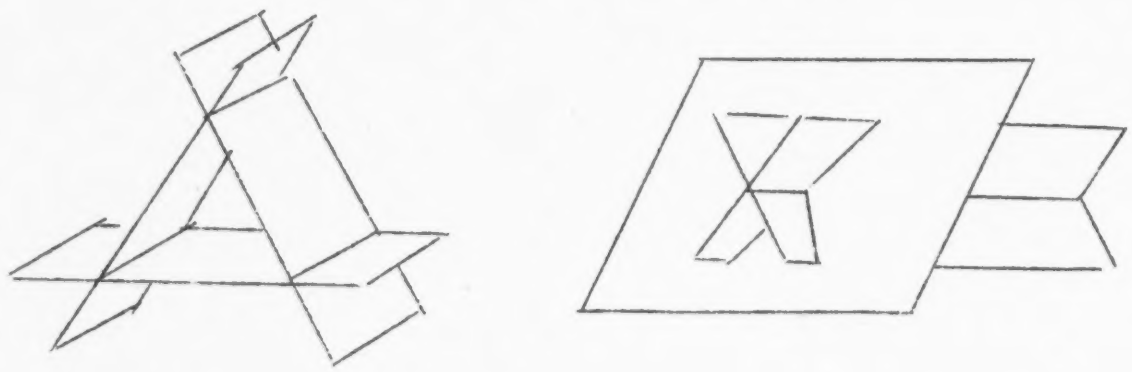
| 44 (cont'd) |  | | |

SUGGESTED BASIC ASSIGNMENT:

MRev... P. 347: Ex. 1, 2, 3, 4
M..... P. 324: Ex. 1, 2, 3, 4

45 Review

SUGGESTED BASIC ASSIGNMENT:

MRev... Review Exercises P. 352:
Ex. 1, 2, 3, 4, 5, 9ab
M..... Review Exercises P. 328:
Ex. 1, 2, 3, 4, 5, 9ab

These problems make use of the algebraic representations of both lines and planes. Great caution must be exercised by the student in keeping the two distinct.

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| 45 (cont'd) | The results in the answer section do not always make use of the obvious, given point in writing the equations for a line. Nevertheless the results are correct and this apparent discrepancy illustrates again the wide choice of expressions which one has available to describe a line in space. Students may wish to work for the answer given by the author, but it should be understood that this is not obligatory. | | |
| 46 | Test | | |

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| | **UNIT SIX** | | |
| | **The Circle** | | |
| | Introduction | | |
| | By some definitions a circle is a conic section; by others, it is not. Refer to Sections 5-1 and 5-2. | | |
| | The word "radius" is used in this text both as a line segment and as a distance. | | |
| 47 | Standard Equation (4-1) | MRev... | 103-104 |
| | | M..... | 91-93 |
| | The General Equation of a Circle (4-2) | MRev... | 106-107 |
| | | M..... | 93-95 |
| | In previous Geometry courses the students have had some work with writing equations of circles where the center and radius is given. They will also be familiar with finding the center and radius of a circle when the equation of a circle is given. A drill would help the teacher discover the students' background on this subject. | | |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 105: Ex. 1f, 2c, 4c, 5ac, 6c, 8<br>M..... P. 93: Ex. 1f, 2c, 4c, 5ac, 6c (and 8 of MRev.)<br>MRev... P. 108: Ex. 1, 11, 13, 16<br>M..... Pp. 95, 96: Ex. 1, 11, 13 (and 16 of MRev.) | | |

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| 48 | **Circle Determined by Three Conditions (4-3)**<br><br>An alternate method to the determinant solution given in the text is suggested as follows:<br>(1) To find the equation of a circle which passes through three given points, find the equations of the perpendicular bisectors of two segments determined by the points.<br>(2) The points of intersection of the perpendicular bisectors is the center of the circle.<br>(3) The radius of the circle will be the distance from the center to one of the given points.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 111: Ex. 1ah, 4ah<br>M..... Pp. 98, 99: Ex. 1ah, 4ah | MRev... 108-111<br>M..... 96-99 |
| 49 | **Symmetry (4-4)**<br><br>**Tangents to a Circle from an External Point (4-5)**<br><br>**Length of a Tangent from a Point Outside a Circle to the Circle (4-6)**<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 117: Ex. 1, 2, 3, 4, 5<br>M..... Pp. 104, 105: Ex. 1, 2, 3, 4, 5 | MRev... 112-114<br>M..... 99-100<br><br>MRev... 114-115<br>M..... 100-101<br><br>MRev... 115-116<br>M..... 102-103 |

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| 50 | Radical Axis (4-7)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 121: Ex. 1ac, 2ab, 3ac<br>M..... P. 108: Ex. 1ac, 2ab, 3ac | MRev... | 117-121<br>M..... 104-108 |
| 51 | Equation of the Tangent to the Circle $x^2 + y^2 = r^2$ at a Point on the Circle (4-8) | MRev... | 122-123<br>M..... 109-110 |
| | Equation of the Tangent to the Circle $x^2 + y^2 = r^2$ when the Slope of the Tangent is Known (4-9)<br>The student should not memorize any special formulas to do the problems in this section. Instead, each problem should be solved using vector methods. | MRev... | 124-126<br>M..... 111-113 |
| | Parametric Equations of the Circle (4-11)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 123, 124: Ex. 1af, 3af, 5af<br>M..... P. 110: Ex. 1af, 3af, 5af<br>MRev... P. 126: Ex. 1a, 2a, 3a<br>M..... P. 113: Ex. 1a, 2a, 3a | MRev... | 132-133<br>M..... 118-120 |
| 52 | Review<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 133, 135: Ex. 1 (a-h inclusive), 3, 5, 11, 16<br>M..... Pp. 120, 121: Ex. 1 (a-h inclusive), 3, 5, 11, 16 | | |
| 53 | Test | | |

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| | UNIT SEVEN<br>The Conics | | |
| 54 | The Conic as a Section of a Cone (5-1)<br>General Definition of a Conic (5-2)<br>The teacher should be aware that when e = 0 the definition fails. But when e approaches zero, the ellipse approaches a circle as a limit. Hence, a circle is a special kind of ellipse.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev...P. 141: Ex. 1, 2, 3, 4, 9, 10, 12<br>M.....P. 127: Ex. 1, 2, 3, 4, 9, 10, 12 | MRev... 138-139<br>M..... 124-127 | |
| 55 | The Parabola<br>Explanation of Terms (5-3)<br>A Geometric Construction of the Parabola (5-4)<br>Simple Equations of the Parabola (5-5)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev...P. 146: Ex. 4, 5, 6, 7a, 8a<br>M..... P. 132: Ex. 4, 5, 6, 7a, 8a | MRev... 142-145<br>M..... 128-131 | |
| 56 | The Latus Rectum (5-6) | MRev... 146-147<br>M..... 132-133 | |

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| 56 (cont'd) | SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 147, 148: Ex. 3, 4<br>M..... Pp. 133, 134: Ex. 3, 4 | | |
| 57 | Parabolic Arch (5-7)<br>Parametric Equations of the Parabola (5-8)<br>Applications of Parabolic Curves (5-9)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 151: Ex. 1, 2, 3<br>M..... P. 137: Ex. 1, 2, 3 | MRev... | 148-151<br>M..... 134-137 |
| 58 | Simple Equations of the Ellipse (5-10)<br>Compare the definition of the ellipse with that of the parabola, noting the value of "e". | MRev... | 153-154<br>M..... 139-140 |
| 59 | Simple Equations of the Ellipse (continued)<br>Develop the simple equation of the ellipse. (The set of calculus projectuals contains a transparency on this topic. However, it is not based on the definition of the ellipse which is used in our text.)<br>Apply $b^2 = a^2 - c^2$ and $c = ae$ to the writing of simple equations of the ellipse. | MRev... | 154-156<br>M..... 140-143 |

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| 59 (cont'd) | SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 157: Ex. 1, 4, 5ace<br>M..... Pp. 142, 143: Ex. 1, 4, 5ace | | |
| 60 | Explanation of Terms for the Ellipse (5-11) | MRev... | 157-158 |
| | | M..... | 143-144 |
| | The Focal Radii (5-12) | MRev... | 159 |
| | | M..... | 145 |
| | SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 158, 159: Ex. 1, 3, 5, 7, 9<br>M..... P. 144: Ex. 1, 3, 5, 7, 9<br>MRev... P. 159: Ex. 2<br>M..... P. 145: Ex. 2 | | |
| 61 | Discussion of the Equation $Ax^2 + Cy^2 + F = 0$ where A and C are of Like Sign and are Not Equal to Zero (5-13) | MRev... | 160-162 |
| | | M..... | 145-147 |
| | Clarify the meaning of "discussion" of an equation. (Refer to Ex. 5-5, MRev... P. 161, M..... P. 147.) | | |
| | Define point ellipse and imaginary ellipse. Special note should also be made at this time concerning the circle as a special ellipse. (Refer to MRev... P. 161, M..... P. 146.) | | |
| | The Latus Rectum of an Ellipse (5-14) | MRev... | 162-163 |
| | | M..... | 148 |
| | SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 163, 164: Ex. 1ab, 2ace, 3ace, 4a, 5<br>M..... P. 149: Ex. 1ab, 2ace, 3ace, 4a, 5 | | |

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| 62 | Construction of an Ellipse (5-15)<br>The Construction in Method 1 is an Application of 5-12<br>Parametric Equations of the Ellipse (5-16)<br>These follow directly from construction method 2.<br>Applications of the Ellipse (5-17)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 163, 164: Ex. 1d, 2f, 3h, 4c<br>M..... P. 149: Ex. 1d, 2f, 3h, 4c<br>MRev... P. 167: Ex. 1ac<br>M..... P. 152: Ex. 1ac | MRev...<br>M.....<br>MRev...<br>M.....<br>MRev...<br>M..... | 165-167<br>150-151<br>167<br>152<br>168<br>152 |
| 63 | Simple Equation of the Hyperbola (5-18)<br>Use the definition of 5-2 to derive the simple equation of the hyperbola.<br>Explanation of Terms for the Hyperbola (5-19)<br>Students should become adept at the discussion of the simple hyperbola.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 170: Ex. 4<br>M..... P. 154: Ex. 4<br>MRev... P. 175: Ex. 1be, 2acei<br>M..... P. 159: Ex. 1be, 2acei | MRev...<br>M.....<br>MRev...<br>M..... | 168-170<br>152-154<br>171-173<br>155-157 |

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| 64 | Asymptotes of Hyperbola (5-20)<br>The definition of the asymptote as given in the text should be stressed in preference to such informal and incorrect phrases as "asymptote never touches the curve". This may be true for the asymptotes of a hyperbola; however, it is not true for asymptotes of such curves as $y = e^x \sin x$.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 175: Ex. 1df, 2flm, 3<br>M..... P. 159: Ex. 1df, 2flm, 3 | MRev... | 173-175<br>M..... 157-159 |
| 65 | The Focal Radii (5-21)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 178: Ex. 1, 2<br>M..... P. 161: Ex. 1, 2 | MRev... | 176-177<br>M..... 160-161 |
| 66 | Discussion of the Equation $Ax^2 + Cy^2 + F = 0$ where $A \neq C \neq 0$, and A and C are Opposite in Sign (5-22)<br>The Latus Rectum of the Hyperbola<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 180: Ex. 2, 3<br>M..... P. 164: Ex. 2, 3 | MRev... | 178-179<br>M..... 161-163<br>MRev... 179-180<br>M..... 163-164 |
| 67 | Conjugate Hyperbolas (5-24)<br>Equilateral Hyperbolas (5-25) | MRev... | 181<br>M..... 164<br>MRev... 181<br>M..... 164-165 |

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| 67 (cont'd) | SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 181: Ex. 1abd, 2abceoqtx<br>M..... P. 165: Ex. 1abd, 2abceoqtx | | |
| 68 | Construction of the Hyperbola (5-26) | MRev... | 183-184<br>M..... 166-167 |
| | Applications of the Hyperbola (5-27) | MRev... | 184<br>M..... 167 |
| | Parametric Equations of a Hyperbola (5-28) | MRev... | 184<br>M..... 167 |
| | SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 185: Ex. 1ad, 2ad<br>M..... P. 168: Ex. 1ad, 2ad | | |
| 69 | Summary and Review (5-29)<br>All the problems in the review exercises are good examples of the concepts of this unit. The assignment below represents only a minimum.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 186: Ex. 1abd, 2abceg, 4, 9<br>M..... P. 169: Ex. 1abd, 2abceg, 4, 9 | MRev... | 185-188<br>M..... 168-169 |
| 70 | Test | | |

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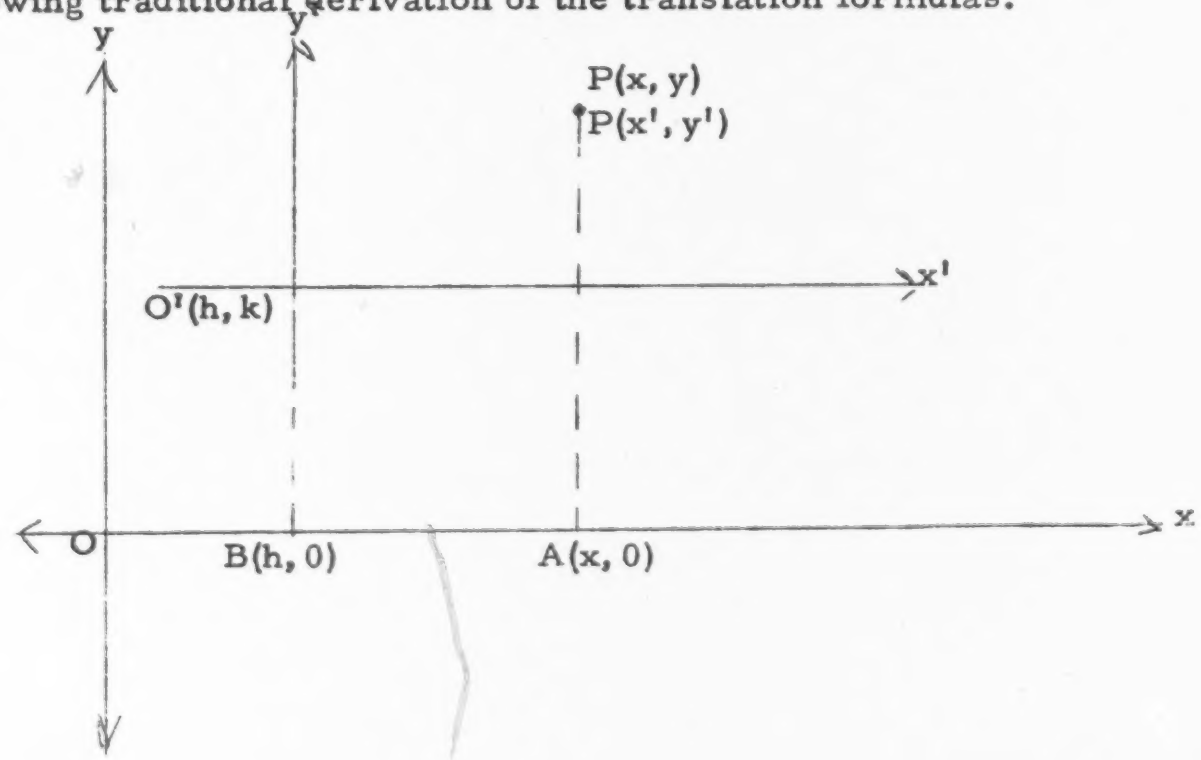
| | UNIT EIGHT<br>Transformation of the Axes | | |
| 71 | Introduction (6-1) | MRev... 192<br>M..... 174 | |
| | Translation of the Axes (6-2) | MRev... 192-195<br>M..... 174-176 | |

The text uses a vector proof to obtain the formulas for the translation of the axes. However, the traditional scalar proof is used when the formulas for the rotation of the axes are derived. For consistency, the teacher may wish to use the following traditional derivation of the translation formulas.



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| 71 (cont'd) | THEOREM: Let x-y be a rectangular coordinate system and let x'-y' be a second rectangular coordinate system which is a translation of the first system. If the coordinates of a point P relative to the x-y system are P(x, y) and the coordinates of P relative to the x'-y' system are P(x', y'), then<br>$x = x' + h$<br>and $y = y' + k$<br>where O'(h, k) are the coordinates of the center of the x'-y' system with respect to the x-y system.<br>PROOF: Project P(x, y) onto the x-axis. Let A be the image of P in the x-axis where A has coordinates (x, 0). Similarly, the image of O' in the x-axis is B with coordinates (h, 0), by the definition of a translation. Thus,<br>$OA = OB + BA$<br>$x = h + x'$<br>or $x' = x - h.$<br>Similarly, $y' = y - k.$<br>Q. E. D.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 194, 195: Ex. 1ace, 2aeg, 3acde<br>M..... Pp. 176, 177: Ex. 1ace, 2aeg, 3acde | | |

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| 72 | Simplification of the Equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$, Where Both A and C are Not Zero (6-3)<br><br>The formulas developed in this section should <u>not</u> be memorized. The student should learn the general procedure and apply it in each example. For instance, to eliminate the linear terms in<br>$4x^2 - y^2 + 24x + 2y + 36 = 0$<br><br>The student should replace x by $x' + h$ and y by $y' + k$ and determine the values of h and k which transforms the equation to one of the form<br>$Ax'^2 + Cy'^2 + F = 0.$<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev...P. 199: Ex. 2, 4, 8, 15<br>M..... P. 181: Ex. 2, 4, 8, 15 | | MRev... 195-200<br>M..... 177-182 |
| 73 | The Standard Equations of the Conics (6-4)<br><br>The teacher should develop in class:<br>(1) the standard equations for the parabola<br>(2) either the standard equation of the ellipse or the standard equation of the hyperbola.<br>Emphasize that the point C with coordinates (h, k) is the <u>center</u> of the ellipse or hyperbola, but that point C with coordinates (h, k) is the <u>vertex</u> of the parabola in standard form. The students should be able to complete charts similar to the following for each conic. | | MRev... 200-205<br>M..... 182-187 |

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| 73 (cont' d) | A DISCUSSION OF THE HYPERBOLA IN STANDARD FORM | | |

| Equation | Center | Transverse Axis | Eccentricity | Vertices |
|---|---|---|---|---|
| $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ | $c(h, k)$ | $x = h$ | $e = \frac{c}{a}$ or $e = \frac{\sqrt{a^2+b^2}}{a}$ | $V_1(h+a, k)$ $U_2(h-a, k)$ |

| Axis of Symmetry | End Points of Minor Axis | Foci | Directrices |
|---|---|---|---|
| $y = k$ | $B_1(h, k+b)$ $B_2(h, k-b)$ | $F_1(h+a\epsilon, k)$ $F_2(h-a\epsilon, k)$ | $x = h + \frac{a}{\epsilon}$ $x = h - \frac{a}{\epsilon}$ |

| Asymptotes | End Points of Latera Recta |
|---|---|
| $\frac{x-h}{a} \pm \frac{y-k}{b} = 0$ | $\angle_1 (k + a\epsilon, k \pm b^2/a)$ $\angle_2 (k - a\epsilon, k \pm b^2/a)$ |

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| 73 (cont'd) | The student is not expected to fill out all columns for the general equation form of the hyperbola. However, in any given problem, all entries should be completed for the discussion.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 205: Ex. 2acegikmoqsu<br>M..... P. 187: Ex. 2acegikmoqsu | | |
| 74 | Rotation of the Axes (6-5)<br><br>Students familiar with matrices may wish to write formulas (6-17) to (6-20) as<br><br>$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$<br><br>Refer to appendix in the text.<br>The mnemonic device<br><br>$\begin{array}{c\|cc} & x' & y' \\ \hline x & \cos\theta & -\sin\theta \\ y & \sin\theta & \cos\theta \end{array}$<br><br>is useful in remembering linear equations (6-15, 14, 15, 16)<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... Pp. 209, 210: Ex. 1acd, 2ace<br>M..... Pp. 191, 192: Ex. 1acd, 2ace | MRev... | 205-210<br>M..... 187-192 |

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| 75 | Simplification of the Equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ Where $B \neq 0$ (6-6)<br><br>The teacher should develop equation (6-26) very carefully in class. Make sure the students understand the sign convention for this formula thoroughly. Prove in class that choosing the signs of cos 2θ and ctn 2θ identically guarantees that the xy-term can always be eliminated by a rotation through a positive acute angle. The student should not memorize formula (6-26), but should know<br>$\cot 2\theta = \frac{A-C}{B}$<br>and<br>$\sin \theta = \sqrt{\frac{(1 - \cos 2\theta)}{2}}$ and $\cos \theta = \sqrt{\frac{(1 + \cos 2\theta)}{2}}$ .<br><br>The teacher should develop methods for simplifying the general quadratic equation in two variables for only those cases which have an xy term but no linear term.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 215: Ex. 1, also eliminate the xy-term and discuss the equation xy = 1<br>M..... P. 197: Ex. 1, also eliminate xy-term and discuss the equation xy = 1 | MRev... 210-212<br>M..... 192-194 | |
| 76 | Simplification of the Equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ Where $B \neq 0$ (6-6) (Continued from Lesson 75) | MRev... 212-215<br>M..... 194-197 | |

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| 76 (cont'd) | Develop methods for simplifying the general quadratic equation in two variables in which both the xy-term and linear terms appear.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 215: Ex. 7<br>M..... P. 197: Ex. 7 | | |
| 77 | Invariants of the Second Degree Equation (6-7) | | MRev... 216-218<br>M..... 198-200 |
| 78 | The Characteristic $4AC - B^2$ and the Discriminant $\Delta$ (6-8)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... Pp. 221,222: Ex. 1, 3, 4, 5, 6, 9<br>M..... P. 202: Ex. 1, 3, 4, 5, 6, 9 | | MRev... 218-220<br>M..... 200-202 |
| 79 | Review | | |
| 80 | Test on Chapter 6 | | |

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| | **UNIT NINE** | | |
| | **Polar Coordinates** | | |
| 81 | Polar Coordinates of a Point (7-2) | MRev... | 228-230 |
| | | M..... | 209-211 |
| | Emphasize that the point $P(r, \theta)$ also has the coordinates $P(r, \theta + 2k\pi)$ and $P(-r, \theta + (2k + 1)\pi)$, where k is integral. Hence, there does <u>not</u> exist a 1-1 correspondence between the set of points in the plane and the ordered pairs of real numbers thought as polar coordinates. (Refer to top of page 230.) This leads to strange occurrences (see Section 7-12). | | |
| | The Relation Between Polar Coordinates and the Cartesian Coordinates (7-3) | MRev... | 230-234 |
| | | M..... | 211-215 |
| | The teacher should note carefully that the problem of changing the form of an equation from polar coordinates to rectangular coordinates, or conversely. This is equivalent to proving that the set of points defined by polar coordinates is equal to the set of points defined by cartesian coordinates. Since $A = B$ if and only if $A \subseteq B$ <u>and</u> $B \subseteq A$, the student must show <u>both</u> the rectangular form and polar form of an equation can be changed into the other <u>exactly</u>. | | |
| | SUGGESTED BASIC ASSIGNMENT: | | |
| | MRev... P. 230: Ex. 2abef<br>M..... P. 211: Ex. 2abef<br>MRev... P. 234: Ex. 1aeg, 2abcg<br>M..... P. 215: Ex. 1aeg, 2abcg | | |

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| 82 | The Line (7-4)<br><br>Consider only the cases of the horizontal line, vertical line, and a line through the pole. There is no need to discuss the general line in polar coordinates.<br><br>The Circle (7-5)<br><br>Discuss the following cases:<br>(1) a circle with center at the pole and radius r<br>(2) a circle with center on the polar axis and passing through the pole<br>(3) a circle with center on the 90° axis and passing through the pole.<br><br>Do not hold students responsible for the general case of the circle.<br><br>SUGGESTED BASIC ASSIGNMENT:<br><br>MRev... P. 236: Ex. 1, 2, 6, 15<br>M..... P. 217: Ex. 1, 2, 6<br>MRev... Pp. 238, 239: Ex. 2ac, 4abd, 5abc<br>M..... Pp. 218, 219: Ex. 2ac, 4abd, 5abc | MRev... 234-236<br>M..... 215-217<br><br>MRev... 236-239<br>M..... 217-219 | |
| 83 | Intercept Points (7-6)<br><br>Symmetry (7-7)<br><br>Observe that in rectangular coordinates the graph of $y = f(x)$ is symmetrical with respect to the y-axis if and only if $f(-x) = f(x)$. However, in polar coordinates, this condition is not sufficient. | MRev... 239<br>M..... 219-220<br><br>MRev... 239-243<br>M..... 220-223 | |

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| 83 (cont'd) | For example, if $f(r,\theta) \neq f(r, -\theta + 2k\pi)$, symmetry with respect to the polar axis may exist. Further tests are necessary. We must check to see whether or not $f(r,\theta) = f(-r, -\theta + [2k+1]\pi)$. If this is the case, then the graph of $f(r,\theta) = 0$ is indeed symmetrical with respect to the polar axis. This is one of the consequences of the fact that there does not exist a 1-1 correspondence between the points in the plane and pairs of polar coordinates. Similar remarks apply to symmetry with respect to the $\frac{\pi}{2}$ -axis and with respect to the pole.<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev.. Pp. 247, 248: Discuss the symmetry of the graphs of the equations in exercises ladg(ah)<br>M..... Pp. 227, 228: Discuss the symmetry of the graphs of the equations in exercises ladg(ah) | | |
| 84 | Extent (7-8) | MRev... 243<br>M..... 223-224 | |
| | Sketching Curves Representing $p = f(\theta)$ (7-9)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 248: Ex. 2ace<br>M..... P. 228: Ex. 2ace | MRev... 244<br>M..... 224-227 | |
| 85 | The Conic (7-10)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 251: Ex. 1a, 2a<br>M..... P. 231: Ex. 1a, 2a | MRev... 249<br>M..... 229-231 | |

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| 86 | Locus Problems (7-11)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 253: Ex. 2<br>M..... P. 233: Ex. 2 | MRev... | 251<br>M..... 232-233 |
| 87 | Intersection of Curves in Polar Coordinates (7-12)<br>SUGGESTED BASIC ASSIGNMENT:<br>MRev... P. 258: Ex. 1, 3, 5<br>M..... P. 239: Ex. 1, 3, 5 | MRev... | 253<br>M..... 233-238 |
| 88 | Review | MRev... | 259<br>M..... 239 |
| 89 | Test | | |